## PROP. VII. THEOR. V.

Si pro $e + fz^{n} + gz^{2n} +$ &c. ſcribatur R ut ſupra, & in Curvæ alicujus Ordinata $z^{\theta \pm n\sigma} R^{\lambda \pm \tau}$ maneant quantitates datæ $\theta$, $n$, $\lambda$, e, f, g, &c. & pro $\sigma$ ac $\tau$ ſcribantur ſucceſſive numeri quicunq; integri : & ſi detur area unius ex Curvis quæ per Ordinatas innumeras ſic prodeuntes deſignantur ſi Ordinatæ ſunt duorum nominum in vinculo radicis, vel ſi dentur areæ duarum ex Curvis ſi Ordinatæ ſunt trium nominum in vinculo radicis, vel areæ trium ex Curvis ſi Ordinatæ ſunt quatuor nominum in vinculo radicis, & ſic deinceps in infinitum : dico quod dabuntur areæ curvarum omnium. Pro nominibus hic habeo terminos omnes in vinculo radicis tam deficientes quam plenos quorum indices dignitatum ſunt in progreſſione arithmetica. Sic ordinata $\sqrt{a^{4} - ax^{3} + x^{4}}$ ob terminos duos inter $a^{4}$ & $-ax^{3}$ deficientes pro quinquinomio haberi debet. At $\sqrt{a^{4} + x^{4}}$ binomium eſt & $\sqrt{a^{4} + x^{4} - \frac{x^{8}}{a^{4}}}$ trinonium, cum progreſſio jam per majores differentias procedat. Propoſitio vero ſic demonſtratur.

### *CAS.* I.

Sunto Curvarum duarum Ordinatæ $pz^{\theta-1} R^{\lambda-1}$ & $qz^{\theta+n-1} R^{\lambda-1}$, & areæ pA & qB, exiſtente R quantitate trium nominum $e + fz^{n} + gz^{2n}$. Et cum per Prop.



Prop. III. ſit $z^{\theta}R^{\lambda}$ area curvæ cujus Ordinata eſt $\theta e \begin{smallmatrix}+\theta\\+\lambda n\end{smallmatrix} fz^{n} \begin{smallmatrix}+\theta\\+2\lambda n\end{smallmatrix} gz^{2n}$ in $z^{\theta-1}R^{\lambda-1}$, ſubduc Ordinatas & areas priores de area & Ordinata poſteriori, & manebit $\begin{smallmatrix}\theta e\\-p\end{smallmatrix} \begin{smallmatrix}+\theta\\+\lambda n\end{smallmatrix} fz^{n} \begin{smallmatrix}+\theta\\+2\lambda n\end{smallmatrix} gz^{2n}$ in $z^{\lambda-1} R^{\lambda-1}$ Ordinata nova Curvæ,&
$-qz^{n}$

$z^{\theta}R^{\lambda} - pA - qB$ ejuſdem area. Pone $\theta e = p$ & $\theta f + \lambda n f = q$ & Ordinata evadet $\begin{smallmatrix}\theta\\+2\lambda n\end{smallmatrix} gz^{2n}$ in $z^{\theta-1}R^{\lambda-1}$, & area $z^{\theta}R^{\lambda} - \theta eA - \theta fB - \lambda n fB$. Divide utramq; per $\theta g + 2\lambda n g$, & aream prodeuntem dic C, & aſſumpta utcunq; r, erit rC area Curvæ cujus Ordinata eſt $rz^{\theta+2n-1}R^{\lambda-1}$. Et qua ratione ex areis pA & qB aream rC Ordinatæ $rz^{\theta+2n-1} R^{\lambda-1}$ congruentem invenimus, licebit ex areis qB & rC aream quartam puta sD, ordinatæ $sz^{\theta+3n-1}R^{\lambda-1}$ congruentem invenire, & ſic deinceps in infinitum. Et par eſt ratio progreſſionis ab areis B & A in partem contrariam pergentis. Si terminorum $\theta$, $\theta + \lambda n$, & $\theta + 2\lambda n$ aliquis deficit & ſeriem abrumpit, aſſumatur area pA in principio progreſſionis unius & area qB in principio alterius, & ex his duabus areis dabuntur areæ omnes in progreſſione utraque. Et contra, ex aliis duabus areis aſſumptis fit regreſſus per analyſin ad areas A & B, adeo ut ex duabus datis cæteræ omnes dentur. Q. E. O. Hic eſt caſus Curvarum ubi ipſius z index $\theta$ augetur vel diminuitur perpetua additione vel ſubductione quantitatis $n$. Caſus alter eſt Curvarum ubi index $\lambda$ augetur vel diminuitur unitatibus.